Using the no-signaling condition for constraining the nonidealness of a Stern–Gerlach set-up

2009 J. Phys. A: Math. Theor. 42 085301

(http://iopscience.iop.org/1751-8121/42/8/085301)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **42** (2009) 085301 (8pp) doi:10.1088/1751-8113/42/8/085301

Using the no-signaling condition for constraining the nonidealness of a Stern–Gerlach set-up

Dipankar Home and Alok Kumar Pan

CAPSS, Department of Physics, Bose Institute, Salt Lake, Calcutta 700 091, India

E-mail: dhome@bosemain.boseinst.ac.in and apan@bosemain.boseinst.ac.in

Received 22 October 2008, in final form 15 December 2008
Published 22 January 2009
Online at stacks.iop.org/JPhysA/42/085301

Abstract
On the basis of a variant of the EPR-Bohm example, we show that the no-signaling condition can be employed as a useful tool for deriving a constraint on a suitably defined measure of the 'nonidealness' of a Stern–Gerlach (SG) set-up. In this demonstration, a key ingredient is provided by the characteristics of the exact solution of the time-dependent Schrödinger equation as applied to a most general SG set-up.

PACS number: 03.65.Ta

1. Introduction

Spurred on by Bell's seminal work [1] related to the EPR-Bohm example [2, 3], the study of quantum mechanical correlations between the results of measurements on the spatially separated particles in the entangled states has become a vibrant research enterprise. Among its various ramifications, a particularly curious feature is that, in spite of an underlying 'nonlocality' embodied in the EPR-Bohm-type correlations, the rules of quantum mechanics turn out to ensure that such a 'nonlocality' cannot be used for sending information in a controlled way that may lead to causality paradoxes. The way this no-signaling condition (signal locality) is satisfied by the quantum mechanical formalism for the entangled states has already been the subject of a number of analyses in different forms [4–10]; nevertheless, it is instructive to probe with respect to new types of examples the way the validity of this condition gets ensured, thereby leading to interesting constraints on the operations of certain quantum devices.

It is in the above-mentioned context that we probe in this paper a variant of the EPR-Bohm example that has a special interest because it involves the use of a *nonideal* quantum measurement (namely, by using the most general nonideal Stern–Gerlach (SG) set-up) in which the properties of explicit solutions of the relevant time-dependent Schrödinger equation play a critical role. Then, in the example considered here, we find that even though the Schrödinger

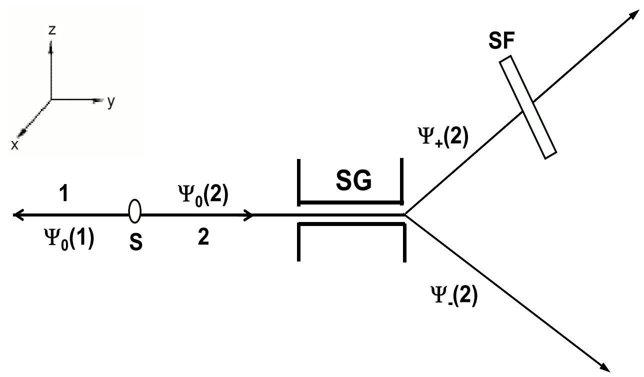

Figure 1. A schematic set-up of the EPR-Bohm type example using a Stern–Gerlach device in one of the two wings of the entangled pairs of particles (see the text for details).

dynamics is intrinsically nonrelativistic, the relevant specifics of the Schrödinger dynamics turn out to be compatible with the no-signaling condition, crucially through a mathematically valid inequality that acts as a *constraint* limiting an appropriately defined measure of the 'nonidealness' of a SG set-up. Before demonstrating this result in section 3 on the basis of an appropriate physical reasoning, we first formulate in the next section the required variant of the EPR-Bohm example, alongside delineating some key features of a nonideal SG set-up that will be used in our argument.

2. The EPR-Bohm example with a nonideal SG set-up

Let us begin with a source emitting EPR-Bohm entangled pairs in spin singlets. In particular, we consider the pairs propagating along opposite directions. The initial total wavefunction is given by

$$|\Psi\rangle_i = \frac{1}{\sqrt{2}}|\psi_0\rangle_1|\psi_0\rangle_2\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right), \tag{1}$$

where the spatial parts $|\psi_0\rangle_1$ and $|\psi_0\rangle_2$ (represented by Gaussian wave packets) correspond to particles 1 and 2 respectively , and the spin part corresponds to the singlet state.

Next, a SG set-up is placed along one of the two wings of the EPR-Bohm pairs, say, for particles 2 moving along the $+y$-axis (figure 1). After passing through the inhomogeneous magnetic field in the SG set-up oriented along, say, the $+z$-axis, these particles belong to the spatially separated wave packets represented by $|\psi_+(\mathbf{x},t)_2|^2$ and $|\psi_-(\mathbf{x},t)_2|^2$. These two wave packets freely propagate along the y–z plane (with equal and opposite momenta of their peaks), corresponding to the spin-up ($|\uparrow\rangle$) and spin-down ($|\downarrow\rangle$) states, respectively.

Consequently, the total wavefunction of the entangled pairs after the particles 2 have gone through the SG magnetic field is given by

$$|\Psi\rangle_{\mathrm{SG}} = \frac{1}{\sqrt{2}}|\psi_0\rangle_1[\psi_-(\mathbf{x},t)_2|\uparrow\rangle_1|\downarrow\rangle_2 - \psi_+(\mathbf{x},t)_2|\downarrow\rangle_1|\uparrow\rangle_2], \tag{2}$$

where $\psi_+(\mathbf{x},t)_2$ and $\psi_-(\mathbf{x},t)_2$ are solutions of the time-dependent Schrödinger equation for the SG set-up, containing the interaction term $H_{\mathrm{int}} = \mu\widehat{\sigma}\cdot B$. The explicit forms of the spatial wavefunctions $\psi_+(\mathbf{x},t)_2$, $\psi_-(\mathbf{x},t)_2$ and the details of the relevant mathematical treatment are given in the appendix, where the corresponding initial wavefunction of particles 2 is taken to be of the following form:

$$\psi_0(\mathbf{x},0)_2 = \frac{1}{\left(2\pi\sigma_0^2\right)^{3/4}}\exp\left(-\frac{\mathbf{x}^2}{4\sigma_0^2} + \mathrm{i}ky\right). \tag{3}$$

Here, the wave packet $|\psi_0(\mathbf{x}, 0)_2|^2$ is peaked at the entry point $(x = y = z = 0)$ of the SG magnetic field region and σ_0 is the initial width of the wave packet.

Now, we focus on the subensemble of particles 2 emerging from the SG set-up that are confined to the upper y–z plane ($y = 0$ to $+\infty$ and $z = 0$ to $+\infty$). These particles are selected out and passed through a spin-flipper (SF) which flips the spin state $|\uparrow\rangle_2$ to $|\downarrow\rangle_2$. The combined state of particles 1 and 2 after this operation is given by

$$|\psi\rangle_{\mathrm{SG+SF}} = \tfrac{1}{2}|\psi_0\rangle_1[|\psi_-(\mathbf{x}, t)\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2 - |\psi_+(\mathbf{x}, t)\rangle_2|\downarrow\rangle_1|\downarrow\rangle_2]. \tag{4}$$

Examining the validity of the no-signaling condition in this case boils down to probing under what condition the expectation value of an arbitrary spin observable pertaining to the particles 1 remains unaffected by the above-mentioned spin-flipping operation on the subensemble of particles 2. For this, we have to first pinpoint the relevant features of a *nonideal* SG set-up, clariyfing precisely the criteria of 'ideal' and 'nonidealness' of a SG set-up.

The usual description of an *ideal* measurement of spin (in this particular case, of the variable $\widehat{\sigma_z}$) using the SG set-up assumes the following conditions to be satisfied:

(A) the wavefunctions $\psi_+(\mathbf{x}, t)_2$ and $\psi_-(\mathbf{x}, t)_2$ are mutually orthogonal. This means that the configuration space distinguishability between the wavefunctions $\psi_+(\mathbf{x}, t)_2$ and $\psi_-(\mathbf{x}, t)_2$ defined in terms of the modulus of their inner product is vanishingly small; i.e.,

$$I = \left| \int_{-\infty}^{+\infty} \psi_+^*(\mathbf{x}, t)_2 \psi_-(\mathbf{x}, t)_2 \, \mathrm{d}^3\mathbf{x} \right| \approx 0. \tag{5}$$

(B) The probability of finding particles with the spin $|\uparrow\rangle_2(|\downarrow\rangle_2)$ in the lower (upper) y–z plane is vanishingly small. Satisfying this condition means that the wave packets $|\psi_+(\mathbf{x}, t)_2|^2$ and $|\psi_-(\mathbf{x}, t)_2|^2$ emerging from the SG set-up get well separated in position space (i.e., they eventually become macroscopically distinct) so that the following conditions hold good:

$$\begin{aligned} |\alpha|^2 &= \int_{x=-\infty}^{+\infty} \int_{y=0}^{+\infty} \int_{z=-\infty}^{0} |\psi_+(\mathbf{x}, t)_2|^2 \, \mathrm{d}^3\mathbf{x} \\ &= \int_{x=-\infty}^{+\infty} \int_{y=0}^{+\infty} \int_{z=0}^{+\infty} |\psi_-(\mathbf{x}, t)_2)|^2 \, \mathrm{d}^3\mathbf{x} \approx 0 \end{aligned} \tag{6}$$

and

$$\begin{aligned} |\beta|^2 &= \int_{x=-\infty}^{+\infty} \int_{y=0}^{+\infty} \int_{z=-\infty}^{0} |\psi_-(\mathbf{x}, t)_2|^2 \, \mathrm{d}^3\mathbf{x} \\ &= \int_{x=-\infty}^{+\infty} \int_{y=0}^{+\infty} \int_{z=0}^{+\infty} |\psi_+(\mathbf{x}, t)_2|^2 \, \mathrm{d}^3\mathbf{x} \approx 1. \end{aligned} \tag{7}$$

At this stage, it is important to note that, depending upon the choices of the relevant parameters (namely, the magnetic field gradient, the interaction time and the initial width of the wave packet), the validity of the condition **A** does *not* automatically ensure the validity of the condition **B**—the latter is, in fact, operationally the key condition for the 'idealness' of the SG set-up when it is used for spin measurement [11]. Hence for defining, in general, a nonideal SG set-up, the question of violation of the condition **B** plays a crucial role.

Note that the quantity I remains unchanged with time after the relevant wave packets $|\psi_+(\mathbf{x}, t)_2|^2$ and $|\psi_-(\mathbf{x}, t)_2|^2$ emerge from the SG magnetic field region. This is because these two wave packets evolve under the *same* unitary evolution; i.e., both of them move freely with equal and opposite momenta of their peaks . In contrast, the quantities $|\alpha|^2$ and $|\beta|^2$ are time dependent and, interestingly, both of them *saturate* to *time-independent constant values* at a certain time after emerging from the SG set-up (see the appendix for the details of how this 'saturation' occurs as a consequence of a rigorous solution of the relevant time-dependent Schrödinger equation).

Thus, in order to appropriately characterize the most general *nonidealness* of a SG set-up, it is necessary to use a measure of the 'nonidealness' that can encapsulate the features associated with the parameters $|\alpha|^2$, $|\beta|^2$ given by equations (6) and (7). A convenient choice for this purpose is a quantity defined in the following way:

$$M(t) = \int_{-\infty}^{+\infty} \sqrt{|\psi_+^*(\mathbf{x},t)_2|^2\,|\psi_-(\mathbf{x},t)_2|^2}\;\mathrm{d}^3\mathbf{x} \tag{8}$$

which denotes the position space overlap between the oppositely moving wave packets corresponding to $|\psi_+(\mathbf{x},t)_2|^2$ and $|\psi_-(\mathbf{x},t)_2|^2$.

Note that the parameter $M(t)$ is, in general, time dependent, but *saturates* to a time-independent value M_s after a certain time $t = t_s$ depending upon the values of the relevant parameters in the SG set-up. The lower and upper bounds of M_s are 0 and 1, respectively.

Hence, for an ideal SG set-up we have $I \approx 0$ and $M_s \approx 0$. But, by choosing the values of the relevant parameters one can ensure nonzero appreciable values of both quantities I and M_s. Therefore, the most *general* type of *nonideal* SG set-up is characterized by the conditions $I \neq 0$ and $M_s \neq 0$. Then, in the context of the EPR-Bohm set-up, the following question immediately suggests itself: is there a relationship between I and M_s, or, a bound to the value of M_s in a most general nonideal SG set-up that can be related to the no-signaling condition?

That such a constraint can indeed be obtained is demonstrated in this paper by showing that M_s has to be always greater than or equal to I; otherwise the no-signaling condition (signal locality) would be violated.

3. Constraint on the SG nonidealness from the no-signaling condition

In the variant of the EPR-Bohm example we consider using the most general *nonideal* SG set-up, a subensemble of particles 2 emerging from the SG magnetic field that are confined to the upper y–z plane (i.e., $y \to 0$ to $y \to +\infty$ and $z = 0$ to $z \to +\infty$) is selected out and passed through a spin-flipper (SF). Given this scenario, our analysis proceeds as follows.

In the first stage of the argument, we consider what would happen if the relevant parameters could be adjusted such that the inner product (I) has a *nonzero* value, while the position space overlap M_s is *vanishingly small*, i.e., $I \neq 0$ but $M_s \approx 0$. Such a condition would operationally mean that a negligibly small number of particles with spin $|{\downarrow}\rangle_2(|{\uparrow}\rangle_2)$ will be present in the upper (lower) y–z plane. Now, suppose in such a situation, the particles 2 in the upper y–z plane are selected out and passed through the SF, with their spin state $|{\uparrow}\rangle$ flipped to the spin state $|{\downarrow}\rangle$.

Then, the expectation value of an arbitrary spin observable A pertaining to the particles 1 in the other wing of the EPR-Bohm pairs can be written as follows by using equation (4) ,

$$\begin{aligned}\langle\psi|A|\psi\rangle_{\mathrm{SG+SF}} = \tfrac{1}{2}[&\langle{\uparrow}|A|{\uparrow}\rangle_1|\psi_-(\mathbf{x},t)_2|^2 + \langle{\downarrow}|A|{\downarrow}\rangle_1|\psi_+(\mathbf{x},t)_2|^2\\ &+ [_2\langle\psi_-(\mathbf{x},t)|\psi_+(\mathbf{x},t)\rangle_2\langle{\uparrow}|A|{\downarrow}\rangle_1].\end{aligned} \tag{9}$$

On the other hand, without subjecting the particles 2 in the upper y–z plane to the spin-flipping operation, the expectation value of the above observable, as calculated by using equation (2), is given by

$$\langle\psi|A|\psi\rangle_{\mathrm{SG}} = \tfrac{1}{2}[\langle{\uparrow}|A|{\uparrow}\rangle_1|\psi_-(\mathbf{x},t)_2|^2 + \langle{\downarrow}|A|{\downarrow}\rangle_1|\psi_+(\mathbf{x},t)_2|^2]. \tag{10}$$

Hence, it is evident from equations (9) and (10) that in the supposed situation where the quantity M_s could be considered negligibly small ($\approx$0), along with a finite nonzero value of I, there would be a violation of the no-signaling condition—a violation that may be quantified by a parameter defined in the following way:

$$\Delta = [\langle\psi|A|\psi\rangle_{\mathrm{SG+SF}} - \langle\psi|A|\psi\rangle_{\mathrm{SG}}] = I[\langle\uparrow|A|\downarrow\rangle_1]. \tag{11}$$

It then follows that the maximum value of Δ would be given by $\Delta_{\max} = I$.

Here note that, in order to be compatible with the no-signaling condition, the realizability of a nonideal SG set-up where $I \neq 0$ and $M_s \approx 0$ must be *ruled out*. Next, considering the most general nonideal situation where both the quantities I and M_s are appreciably nonzero (i.e., $I \neq 0$ and $M_s \neq 0$), let us examine whether a bound to the value of M_s can be obtained from the no-signaling condition. For this, we adopt the following strategy for formulating the relevant argument.

To start with, we note that the above estimation of the value of $\Delta_{\max}$ by taking $M_s \approx 0$ is obviously erroneous if the quantity M_s has a non-negligible value. Then a plausible measure of the error involved in such an estimation is the position space overlap parameter itself. Consequently, for the no-signaling condition to be satisfied, this error *cannot* be smaller than the value of $\Delta_{\max}$. This lends itself to the implication that there has to be a *lower bound* to the value of M_s, i.e., for all possible choices of the relevant parameters, any given nonideal SG set-up with finite nonzero values of both quantities M_s and I must satisfy the following inequality:

$$M_s \geqslant I. \tag{12}$$

In other words, the validity of the no-signaling condition in this situation gets related to the following mathematical inequality:

$$\int_{-\infty}^{+\infty} |\psi_+(\mathbf{x}, t)_2||\psi_-(\mathbf{x}, t)_2|\, \mathrm{d}^3\mathbf{x} \geqslant \left| \int_{-\infty}^{+\infty} \psi_+^*(\mathbf{x}, t)_2 \psi_-(\mathbf{x}, t)_2\, \mathrm{d}^3\mathbf{x} \right|. \tag{13}$$

We stress that this is the most *general constraint* on the 'nonidealness' of the SG set-up that can be obtained from the no-signaling condition. While the mathematical validity of the above inequality can be seen from the properties of complex functions, it is interesting to note that the physical condition of compatibility with the no-signaling condition in the EPR-Bohm example warrants the inevitability of such a constraint.

Note that, as a particular case, it follows that the no-signaling condition rules out the realizability of a SG set-up such that $I \neq 0$ and $M_s \approx 0$. There is, of course, another particular case of 'nonidealness', namely, corresponding to $I \approx 0$ and $M_s \approx 0$ which, obviously, does not lead to any inconsistency with the no-signaling condition, as is evident from equation (13).

4. Concluding remarks

The power of the no-signaling condition in giving rise to specific bounds on various types of quantum operations, such as the limits on the fidelity of quantum cloning machines and on quantum state discriminations, has been demonstrated in a number of ways [15]. Our present work complements these studies from a somewhat different perspective, namely, by linking the no-signaling condition with a constraint relation governing an archetypal example of quantum measurement of spin provided by the SG set-up. This suggests the possibility of more uses of the no-signaling condition for probing the bounds inherent in the quantum mechanical modeling of other specific measurement processes—a line of investigation which, in conjunction with the studies made to obtain from the no-signaling condition limits on the possible extensions of quantum mechanics [16], may lead to some interesting restrictions on generalizations of the quantum theory of measurement. This is currently being studied.

Acknowledgments

AKP acknowledges helpful discussions related to this work during his visits to the Perimeter Institute, Canada; Centre for Quantum Technologies, National University of Singapore, and Benasque Centre for Science, Spain. DH is grateful to Paul Davies and John Corbett for interactions that stimulated this work. DH thanks the Centre for Science and Consciousness, Kolkata for support. AKP acknowledges the Research Associateship of Bose Institute, Kolkata.

Appendix

For the sake of completeness, here we give a concise presentation of the ingredients of the quantum mechanical treatment of the SG set-up as relevant to our present paper, while the analyses of the SG set-up have been discussed in various contexts [3, 11, 12]. A beam of *x-polarized* spin-1/2 neutral particles, say, neutrons, passing through the SG magnetic field is represented by the total wavefunction $\Psi\,(\mathbf{x}, t = 0) = \psi_0\,(\mathbf{x})\,\chi(t = 0)$. The spatial part $\psi_0(\mathbf{x})$ corresponds to a Gaussian wave packet which is initially peaked at the entry point $(\mathbf{x} = 0)$ of the SG magnet at $t = 0$, given by

$$\psi_0(\mathbf{x}) = \frac{1}{\left(2\pi\sigma_0^2\right)^{3/4}} \exp\left(-\frac{\mathbf{x}^2}{4\sigma_0^2} + \mathrm{i}\mathbf{k}\cdot\mathbf{x}\right), \tag{A.1}$$

where σ_0 is the initial width of the wave packet. The wave packet moves along the +ve y-axis with the initial group velocity v_y and the wave number $k_y = \frac{mv_y}{\hbar}$. Note that the initial spin state is given by $\chi(t = 0) = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + |\downarrow\rangle_z)$ where $|\uparrow\rangle_z, |\downarrow\rangle_z$ are eigenstates of the spin observable σ_z. The inhomogeneous magnetic field (localized between $y = 0$ and $y = d$) is directed along the +ve z-axis. As the wave packet propagates through the SG magnet, in addition to the $+\widehat{y}$-axis motion, the particles gain velocity with the magnitude v_z along the $\widehat{z}$-axis due to the interaction of their spins with the inhomogeneous magnetic field during the time interval τ.

Here the interaction Hamiltonian is $H_{\text{int}} = \mu\widehat{\sigma}\cdot\mathbf{B}$ where μ is the magnetic moment of the neutron, $\mathbf{B}$ is the inhomogeneous magnetic field and $\widehat{\sigma}$ is the Pauli spin matrices vector. Then the time evolved total wavefunction at $t = \tau$ (τ is taken to be the transit time of the peak of the wave packet within the SG magnetic field region) after the interaction of spins with the SG magnetic field is given by

$$\begin{aligned}\Psi\,(\mathbf{x}, t = \tau) &= \exp\left(-\frac{\mathrm{i}H\tau}{\hbar}\right)\Psi(\mathbf{x}, t = 0)\\ &= \frac{1}{\sqrt{2}}[\psi_+(\mathbf{x}, \tau)\otimes|\uparrow\rangle_z + \psi_-(\mathbf{x}, \tau)\otimes|\downarrow\rangle_z],\end{aligned} \tag{A.2}$$

where $\psi_+(\mathbf{x}, \tau)$ and $\psi_-(\mathbf{x}, \tau)$ are the two components of the spinor $\psi = \binom{\psi_+}{\psi_-}$ which satisfies the Pauli equation. The inhomogeneous magnetic field is represented by $\mathbf{B} = (-bx, 0, B_0 + bz)$ satisfying the Maxwell equation $\nabla\cdot\mathbf{B} = 0$. The two-component Pauli equation can then be written as two coupled equations for ψ_+ and ψ_-, given by

$$\begin{aligned}\mathrm{i}\hbar\frac{\partial\psi_+}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi_+ + \mu(B_0 + bz)\psi_+ - \mu bx\psi_-\\ \mathrm{i}\hbar\frac{\partial\psi_-}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi_- + \mu bx\psi_+ - \mu(B_0 + bz)\psi_-.\end{aligned} \tag{A.3}$$

The coupling between the above two equations can be removed [13, 14] using the condition $B_0 \gg b\sigma_0$, whence one obtains the following decoupled equations given by:

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\psi_+}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi_+ + \mu\,(B_0 + bz)\,\psi_+ \\ \mathrm{i}\hbar\frac{\partial\psi_-}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi_- - \mu\,(B_0 + bz)\,\psi_-. \end{aligned} \tag{A.4}$$

The solutions of the above two equations are as follows:

$$\begin{aligned} \psi_+\,(\mathbf{x};\tau) &= \frac{1}{\left(2\pi s_\tau^2\right)^{\frac{3}{4}}}\exp\left[-\left\{\frac{x^2 + (y - v_y\tau)^2 + (z - \frac{v_z\tau}{2})^2}{4\sigma_0 s_\tau}\right\}\right] \\ &\quad\times\exp\left[\mathrm{i}\left\{-\Delta_+ + \left(y - \frac{v_y\tau}{2}\right)k_y + k_z z\right\}\right] \\ \psi_-\,(\mathbf{x};\tau) &= \frac{1}{\left(2\pi s_\tau^2\right)^{\frac{3}{4}}}\exp\left[-\left\{\frac{x^2 + (y - v_y\tau)^2 + (z + \frac{v_z\tau}{2})^2}{4\sigma_0 s_\tau}\right\}\right] \\ &\quad\times\exp\left[\mathrm{i}\left\{-\Delta_- + \left(y - \frac{v_y\tau}{2}\right)k_y - k_z z\right\}\right], \end{aligned} \tag{A.5}$$

where

$$\Delta_\pm = \pm\frac{\mu B_0\tau}{\hbar} + \frac{m^2 v_z^2\tau^2}{6\hbar^2}, \qquad v_z = \frac{\mu b\tau}{m}, \qquad k_z = \frac{m v_z}{\hbar} \qquad \text{and} \qquad s_t = \sigma_0\left(1 + \frac{\mathrm{i}\hbar t}{2m\sigma_0^2}\right).$$

Here $\psi_+(\mathbf{x},\tau)$ and $\psi_-(\mathbf{x},\tau)$ representing the spatial wavefunctions at $t = \tau$ correspond to the spin states $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ respectively, with the average momenta $\langle\widehat{p}\rangle_\uparrow$ and $\langle\widehat{p}\rangle_\downarrow$, where $\langle\widehat{p}\rangle_{\uparrow\downarrow} = (0, m v_y, \pm\mu b\tau)$.

Hence after emerging from the SG magnet, the particles corresponding to the wavefunction components $\psi_+(\mathbf{x},\tau)$ and $\psi_-(\mathbf{x},\tau)$ move *freely* along the respective directions $\widehat{n}_+ = v_y\widehat{j} + \frac{\mu b\tau}{m}\widehat{k}$ and $\widehat{n}_- = v_y\widehat{j} - \frac{\mu b\tau}{m}\widehat{k}$ with the *same* group velocity $v = \sqrt{v_y^2 + \left(\frac{\mu b\tau}{m}\right)^2}$ which is fixed by the relevant parameters of the SG set-up and the initial velocity (v_y) of the peak of the wave packet.

Now, the modulus of the inner product I between $\psi_+(\mathbf{x},\tau)$ and $\psi_-(\mathbf{x},\tau)$ is given by

$$I = \exp\left\{-\frac{\mu^2 b^2\tau^4}{8m^2\sigma_0^2} - \frac{2\mu^2 b^2\tau^2\sigma_0^2}{\hbar^2}\right\} \tag{A.6}$$

that is necessarily *zero* for the ideal situation. This inner product is preserved for further time evolution during which the freely evolving wavefunctions at a time t_1 *after* emerging from the SG set-up are given by

$$\begin{aligned} \psi_+(\mathbf{x}, t = \tau + t_1) &= \frac{1}{\left(2\pi s_{t_1+\tau}^2\right)^{3/4}}\exp\left[-\left\{\frac{x^2 + \left(y - v_y(\tau + t_1)\right)^2 + \left(z - \frac{v_z\tau}{2} - v_z t_1\right)^2}{4\sigma_0 s_{t_1+\tau}}\right\}\right] \\ &\quad\times\exp\left[\mathrm{i}\left\{-\Delta_+ + k_y\left(y - \frac{v_y(\tau + t_1)}{2}\right) + k_z(z - \frac{v_z t_1}{2})\right\}\right] \\ \psi_-(\mathbf{x}, t = \tau + t_1) &= \frac{1}{\left(2\pi s_{t_1+\tau}^2\right)^{3/4}}\exp\left[-\left\{\frac{x^2 + (y - v_y(\tau + t_1))^2 + (z + \frac{v_z\tau}{2} + v_z t_1)^2}{4\sigma_0 s_{t_1+\tau}}\right\}\right] \\ &\quad\times\exp\left[\mathrm{i}\left\{-\Delta_- + k_y\left(y - \frac{v_z(\tau + t_1)}{2}\right) - k_z\left(z + \frac{v_z t_1}{2}\right)\right\}\right] \end{aligned} \tag{A.7}$$

where

$$s_{t_1+\tau} = \sigma_0\left(1 + \frac{i\hbar(t_1+\tau)}{2m\sigma_0^2}\right).$$

Note that the wave packets $|\psi_+(\mathbf{x}, t = \tau)|^2$ and $|\psi_-(\mathbf{x}, t = \tau)|^2$ emerging from the SG magnet will move away from each other so that the position space overlap between these two wave packets will be changing with time. The position space overlap parameter $M(t)$ as defined in equation (7) is given by

$$M(t) = \exp\left[-\frac{v_z^2(\tau+2t_1)^2}{8\sigma_{\tau+t_1}^2}\right], \tag{A.8}$$

where $\sigma_{\tau+t_1}$ is the width of the wave packet at the instant $\tau + t_1$. From the above equation, it follows that $M(t)$ *saturates* to a *time-independent value* after a sufficiently large time, the saturated value being given by

$$M_s = \exp\left(-\frac{2v_z^2 m_0^2 \sigma_0^2}{\hbar^2}\right). \tag{A.9}$$

References

[1] Bell J S 1964 *Physics* **1** 195
[2] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[3] Bohm D 1952 *Quantum Theory* (Englewood Cliffs, NJ: Prentice-Hall) pp 593–98
[4] Bohm D 1952 *Quantum Theory* (Englewood Cliffs, NJ: Prentice-Hall) pp 618–19
[5] Eberhard P H 1978 *Nuovo Cimento* B **46** 392
Eberhard P H and Ross R R 1989 *Found. Phys. Lett.* **2** 127
[6] Ghirardi G C, Rimini A and Weber T 1980 *Lett. Nuovo Cimento* **27** 293
[7] Page D N 1982 *Phys. Lett.* A **91** 57
[8] Bussey P J 1982 *Phys. Lett.* A **90** 9
[9] Jordan T F 1983 *Phys. Lett.* A **94** 264
[10] Shimony A 1984 *Proc. 1st Int. Symposium on the Foundations of Quantum Mechanics in the Light of New Technology* ed S Kamefuchi (Tokyo: Physical Society of Japan) pp 225–30
[11] Home D, Pan A K, Ali M and Majumdar A *et al* 2007 *J. Phys. A: Math. Theor.* **40** 13975
[12] Scully M O, Lamb W E Jr and Barut A 1987 *Found. Phys.* **17** 575
Busch P and Schroeck F E 1989 *Found. Phys.* **19** 807
Platt D E 1992 *Am. J. Phys.* **60** 306
Patil S H 1998 *Eur. J. Phys.* **19** 25
Roston G B, Casas M, Plastino A and Plastino A R 2005 *Eur. J. Phys.* **26** 657
[13] Alstrom P, Hjorth P and Muttuck R 1982 *Am. J. Phys.* **50** 697
Singh S and Sharma N K 1983 *Am. J. Phys.* **52** 274
[14] Cruz-Barrios S and Gomez-Camacho J 2000 *Phys. Rev.* A **63** 012101
Potel G, Barranco F, Cruz-Barrios S and Gómez-Camacho J 2005 *Phys. Rev.* A **71** 052106
[15] Gisin N 1998 *Phys. Lett.* A **242** 1
Ghosh S, Kar G and Roy A 1999 *Phys. Lett.* A **261** 17
Barnett S M and Andersson E 2002 *Phys. Rev.* A **65** 044307
Qui D 2002 *Phys. Lett.* A **303** 140
Feng Y, Zhang S, Duan R and Ying M 2002 *Phys. Rev.* A **66** 062313
Pati A K and Braunstein S L 2003 *Phys. Lett.* A **315** 208
Hwang W 2005 *Phys. Rev.* A **71** 062315
[16] Popescu S and Rohrlich D 1994 *Found. Phys.* **24** 379
Simon S, Buzek V and Gisin N 2001 *Phys. Rev. Lett.* **87** 170405
Masanes L, Acin A and Gisin N 2006 *Phys. Rev.* A **73** 012112